Deformation Bands, the LEDS Theory, and Their Importance in Texture Development: Part II. Theoretical Conclusions

D. KUHLMANN-WILSDORF

The facts regarding "regular" deformation bands (DBs) outlined in Part I of this series of articles are related to the low-energy dislocation structure (LEDS) theory of dislocation-based plasticity. They prompt an expansion of the theory by including the stresses due to strain gradients on account of changing selections of slip systems to the previously known dislocation driving forces. This last and until now neglected driving force is much smaller than the components considered hitherto, principally due to the applied stress and to mutual stress-screening among neighbor dislocations. As a result, it permits a near-proof of the LEDS hypothesis, to wit that among all structures which, in principle, are accessible to the dislocations, that one is realized which has the lowest free energy. Specifically, the temperature rises that would result from annihilating the largest DBs amount to only several millidegrees Centigrade, meaning that they, and by implication the entire dislocation structures, are close to thermodynamical equilibrium. This is in stark contrast to the assumption of the presently widespread self-organizing dislocation structures (SODS) modeling that plastic deformation occurs far from equilibrium and is subject to Prigogine's thermodynamics of energy-flow-through systems. It also holds out promise for future rapid advances in the construction of constitutive equations, since the LEDS hypothesis is the principal basis of the LEDS theory of plastic deformation and follows directly from the second law of thermodynamics in conjunction with Newton's third law. By contrast, all other known models of metal plasticity are in conflict with the LEDS hypothesis. In regard to texture modeling, the present analysis shows that Taylor's criterion of minimum plastic work is incorrect and should be replaced by the criterion of minimum free energy in the stressed state. Last, the LEDS hypothesis is but a special case of the more general low-energy structure (LES) hypothesis, applying to plastic deformation independent of the deformation mechanism. It is thus seen that plastic deformation is one of nature's means to generate order, as a byproduct of the entropy generation when mechanical work is largely converted into heat.

I. INTRODUCTION

THE plastic deformation of solids is arguably among the most important phenomena in our lives, let alone in engineering, yet it is among the least understood. This is highly regrettable, especially in regard to technological metals whose deformation is mediated through glide dislocations, because it stymies the development of effective constitutive equations. These must inevitably be based on a thorough understanding of the correlations between straining conditions, resulting deformation structures, and the mechanical properties derived therefrom.

This state of affairs is due to the lack of a unified approach. The foundational, basically irreconcilable publications are due to Taylor[1] and Becker,[2–6] respectively. Taylor proposed the first dislocation-based work-hardening theory simultaneously with his introduction of the dislocation concept into materials science. According to it, on stress application, dislocations virtually instantaneously multiply and move *via* glide into configurations that are in mechanical equilibrium with the applied tractions. The resulting dislocation density is proportional to the square of the resolved shear stress, in agreement with firmly established empirical observation. As Taylor assumed a constant mean free-glide path, he obtained a parabolic stress-strain curve.

Becker's 1925 theory,[2,3,4] by contrast, predates the dislocation concept. He proposed that crystals deform, much like fluids and amorphous materials, *via* thermally activated events. While in noncrystalline materials, these events are place changes of individual atoms, Becker envisaged that in crystals, discrete volume elements would shear by elementary glide steps. These glide events would be triggered whenever statistical thermal activation momentarily and locally generated a critical resolved shear stress. In this view, the apparent one-to-one correlation between applied stress and resulting strain, so familiar from work-hardening curves, results (a) from the extremely steep stress dependence of the activated glide event frequency in the vicinity of the critical stress; and (b) because the flow would soon cease through work-hardening, the nature of which Becker did not attempt to elucidate. Rather, Becker had his theory tested by his pupils Boas[5] and Orowan[6] *via* low-strain creep tests of thin metal wires near room temperature.

Becker's theory is no longer under active consideration, but its successor theory, of self-organizing dislocation structures (SODS), has for the past several years received the lion's share of attention. It is an adaptation of a model by Holt[7] and Staker and Holt[8] to Prigogine's thermodynamics of energy-flow-through systems. SODS modeling has been pioneered by Aifantis and Walgraef[9–13] and is currently most prominently associated with the name of Kubin[14,15] and Devincre and Kubin.[16] According to it, dislocations

D. KUHLMANN-WILSDORF, University Professor of Applied Science, is with the Department of Materials Science and Engineering and the Department of Physics, University of Virginia, Charlottesville, VA 22903.

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"self-organize" *via* statistical, elementary, thermally activated glide events, mostly under the influence of their own stress systems. The resulting structures are believed to be far from thermodynamical equilibrium, and the dislocation motions are assumed to obey the thermodynamics applicable to "energy flow-through" systems, including living organism, for which Prigogine received the Nobel prize.

The historical evolution of work-hardening theory was treated in References 17 and 18. Importantly in this connection, Taylor left the field soon after his 1934 article[19] and never elaborated his theory. Its needed expansion is the low-energy dislocation structure (LEDS) theory that has been systematically developed since 1962.[20] The LEDS theory has recently been summarized in two extensive articles.[18,21] It recognizes the pattern initially proposed by Taylor (the "simple Taylor lattice") as but one of a large range of dislocation equilibrium structures, dubbed LEDSs. By definition, LEDSs are structures in which, essentially instantaneously through glide, near-neighbor dislocations mutually screen their stress fields. Specifically, according to the LEDS theory, Taylor lattices arise when three-dimensional dislocation mobility is low, as in "planar-glide materials." By contrast, deformed "wavy-glide materials" are characterized by "mosaic block structures," *i.e.*, almost perfect mutually rotated volume elements that are delineated by dislocation rotation boundaries. The very widespread occurrence of mosaic block structures was empirically recognized long before the advent of dislocation theory. In fact, it represents the most stable known LEDS morphology. However, the formation of a mosaic block structure requires a modicum of three-dimensional dislocation mobility, which is available in wavy- but not planar-glide materials.

In a nutshell then, the LEDS theory adopts G.I. Taylor's assumption that on stress application, dislocations virtually instantaneously assume low-energy equilibrium positions that on cessation of stress increases are frozen-in except for possible annealing effects. For the remainder the LEDS theory does not include models. Rather it attempts to identify the structures of lowest free energy consistent with deformation conditions that are in mechanical equilibrium with the applied tractions, as demanded by the second law of thermodynamics and Newton's third law. It follows from the above that regardless of specific morphology, glide dislocations form configurations in which their respective long-range stresses are screened. Moreover, also regardless of dislocation pattern, the parallel of Gauss's law of electric fields relative to electric charges applies: "In any dislocation structure at rest, the respective resolved shear stresses cannot exceed the friction stress, τ_0, anywhere along any dislocation line."[20] Thus, dislocations mutually trap into energy troughs from which at least some must be released for further glide to take place, either through bypassing of parallel dislocation segments in Taylor lattices or supercritical bowing of the longest dislocation links in networks such as constitute dislocation cell structures. In this light, workhardening arises because it requires an increasingly high stress to release dislocations from those energy troughs that with increasing dislocation density gradually deepen and narrow. From this derives the already mentioned empirical law that, almost independent of the specific dislocation structure, the flow stress is proportional to the root of the dislocation density, which we already saw to be a feature also of Taylor's work-hardening theory.

Next, as with increasing stress and dislocation density the effect of the friction stress relatively decreases, while the availability of different Burgers vectors increases, transformations of the dislocation patterns take place. In the stress-strain curve, these give rise to the successive work-hardening stages I through IV. And further, unless Bauschinger-type long-range internal stresses intervene, the flow stress does not change with stress reduction or reversal.

Implicit in the outlined assumptions of the LEDS theory is the claim that always the lowest possible free energy of the "system" consisting of deforming tractions and deformed material, including its deformation structures, is approached. This means that for vanishing intrinsic resistance against dislocation motion, *i.e.* for friction stress $\tau_0 = 0$, under the applied load which generated them, the observed dislocation structures are in thermodynamical equilibrium and cannot be removed through heat treatments. The illusion that deformation structures embody excess free energy even while still under applied stress, arises through neglecting Newton's third law, *i.e.* that the tractions are an essential part of the "system."[18,21,22]

As the work done by the tractions is 90 pct converted into heat, plastic deformation is one of nature's most effective ways for increasing entropy. However, as a byproduct, order is generated in the form of those structures that are required for force equilibrium (which might prominently include twins, *e.g.*, in hcp metals, and/or other defects besides or instead of dislocations). Still, as soon as the tractions are reduced, the deformation structures are no longer in equilibrium but do indeed embody excess free energy. In the framework of the LEDS theory, therefore, except for minor readjustments on account of τ_0, dislocation structures can be modified or removed through annealing *only* when the applied forces are smaller than the previously highest.

Only very recently[23,24] has it become clear that the LEDS theory is but a special case of the more general low-energy structure (LES) hypothesis. According to it, plastic deformation of solids always generates those structures that impart the lowest free energy to the system of deformed material and tractions among all that are accessible in principle. By way of illustration, the overall structure of the earth, with its metallic core, shells of ceramics graded according to density, oceans, and atmosphere, is that of minimum free energy under the prevailing force of gravity. If gravity were extinguished, disintegration of the earth into space would increase entropy. Yet, although the atmosphere and the oceans would indeed soon so dissipate, the structure of the solid earth is "frozen-in," much like LEDSs on stress removal, in this case, because of cohesion among chunks of matter. And yet, the earth's ideal minimum free energy configuration is not fully attained, namely, because of "energy flow through" processes. Specifically, the water is collected in the oceans and does not cover the earth uniformly, and the landmasses are not flat but peaked by high mountain ranges. In this light, the equivalent of the SODS theory would apply to continental drift and to the weather, *i.e.*, to deviations from the equilibrium structure that are driven by "energy flow through." Presumably, they proceed in accordance with Prigogine's thermodynamics, while ordinary thermodynamics accounts for the overall structure of

the earth. Similarly, the SODS theory does not apply to the generation of the dislocation deformation structures in metals, but it may perhaps be fruitfully applied to annealing processes.

In agreement with the preceding explanation, the LES hypothesis is believed to apply not to just a few special examples but to deformed systems in general. Specifically, it is claimed that as a quite general rule, while the great preponderance of the work done on solids by deforming tractions is transformed into heat at an overall increase of entropy, some order in the deformed solids is generated as a byproduct. This order is thermodynamically stable but *only* as long as the tractions are in place. On removal of the tractions, that order constitutes excess free energy that can be dissipated at further increase of entropy, mostly as a result of annealing.

Confidence in the LES hypothesis and indirectly in the LEDS theory is greatly strengthened by recent observations on fatigue-cycled lamellar diblock copolymers.[24,25] These form kink bands of a morphology expected from the LES hypothesis[24] and, further, on prolonged cycling as a function of temperature and cycling frequency, always that lamella orientation is being established for which the shear modulus parallel to the fatigue stress is the lowest.[25] This is the expected result from the LES hypothesis, since at peak stress, the observed orientation yields the largest strain at the lowest stress and, hence, lowest stored elastic strain energy. Similarly, Laird and co-workers have already established that the dislocation structures in constant-strain-amplitude fatigue of metals are uniquely related to the fatigue stress and are reversible.[26,27]

In line with the preceding arguments, the decision between the LEDS and SODS theories hinges on whether actual dislocation structures in deformed materials are close to or far from thermodynamical equilibrium, as assumed in the LEDS and SODS theory, respectively. Until now, it seemed impossible to quantitatively decide this point: How does one determine the closeness of approach to equilibrium of observed dislocation structures? In this predicament, regular deformation banding, as experimentally examined in Part I of this article, provides an answer. As will be shown, based thereon the approach to thermodynamical equilibrium is found to be surprisingly close, namely, comparable to the heat energy that would raise the temperature of the material by only $\cong 10^{-2}$ °C. Thereby the decision has been made in favor of the LEDS (and LES) hypothesis.

II. BRIEF OUTLINE OF THE LEDS THEORY

A. *Mutual Dislocation Stress Screening Based on the Second Law of Thermodynamics*

The potential impact of the facts relating to regular deformation bands outlined in Part I on the possible expeditious development of technologically useful constitutive equations is incalculable. In order to appreciate this, a brief outline of the present state of the theory will be helpful, as follows: With the exception of the LEDS theory, the various existing models of dislocation-based crystal plasticity assume that workhardening results from some mutual obstruction among glide dislocations, which is associated with excess energy. In those models, somehow or the other, glide dislocations are supposed to mutually block each others progress, *e.g.*, through pileups (Seeger and co-workers), modified pileups (Mott and Hirsch and co-workers), Lomer-Cottrell or Hirth locks, "hedges" (Kocks), hard zones among soft material in a kind of composite structure (Mughrabi), or simply accumulations of mutually obstructive SODS that are assumed to have formed far from equilibrium subject to Prigogine's Nobel-prize winning thermodynamics of energy flow-through systems (Kubin and Aifantis and co-workers), as already outlined in Section I. However, none of those models attempts to explain more than isolated facets of plasticity, *e.g.*, the slope of stage II or the onset of stage III, and none recognize the existence of Taylor lattices or the prevalence of dislocation rotation boundaries in actual deformation structures.

Through postulating critically important structures that contain excess free energy while subject to stresses well beyond the friction stress, τ_0, at which glide dislocations are highly mobile, all of the enumerated models violate the second law of thermodynamics. The LEDS theory does not suffer this defect. It has been patiently developed, beginning with a 1962 article.[20] In accordance with Section I, the most basic, unvarying concept herein is that glide dislocations are bound to arrange into mutually stress-screened (*i.e.*, to the level of τ_0) dislocation structures, since they are the centers of strong internal stress fields while being mobile under their respective resolved shear stresses in excess of τ_0. Consequently, glide dislocations inevitably are the source of, react to, and interact with shear stresses of whatever origin, including contributions due to their own lines if curved. Thus follows the already mentioned fact that at the position of every glide dislocation line at rest, everywhere and at all times, the respective resolved shear stress cannot exceed the friction stress. If it did, the dislocations would move.

B. *"Similitude"*

In a material of average dislocation density ρ, the average distance between dislocations is bound to compare with $l \cong 1/\sqrt{\rho}$. If the dislocations are arranged in the form of networks consisting of dislocation links between threefold nodes, as is the case in all dislocation cell structures characteristic of work-hardening stages III and IV, l is also a measure of the dislocation link lengths. Always the longest of these would have to bow out supercritically to permit further deformation. Thus, the flow stress in stages III and IV is the Frank–Read stress of always the longest dislocation links, given by

$$\tau \approx \tau_0 + Gb/2l = \tau_0 + \alpha Gb\sqrt{\rho} \qquad [1]$$

with G the shear modulus, b the magnitude of the Burgers vector, and the constant α somewhat smaller than unity. Also as a result of mutual stress screening, the average dislocation line energy is

$$\begin{aligned} U_D &\cong Gb^2\,\{(1-\nu/2)/[4\pi(1-\nu)]\}\,\ln(l/b) \\ &\cong 0.1\,Gb^2\ln(l/b) \end{aligned} \qquad [2]$$

where ν is Poisson's ratio.

Besides effortlessly yielding the very widely observed empirical relationship of Eq. [1], this simple theory further led to the "principle of similitude."[20] According to it, in line with Eq. [1], under otherwise constant conditions an increase of applied stress causes a reciprocal decrease of the scale of the prevailing dislocation structure.

C. *General Expression for the Work-Hardening Coefficient*

The outlined simple facts further lead to a general expression for the work-hardening coefficient, at least within work-hardening stages I through III, while conditions in stage IV may be more complicated. Namely, based on similitude, the average free path length of any newly generated glide dislocations will be some fixed multiple of the average dislocation spacing, *i.e.*,

$$gl \cong g/\sqrt{\rho} \qquad [3]$$

with g the dimensionless "dislocation path parameter." Hence, if the mean free path is limited by, say, the walls in a roughly cubic dislocation cell structure, and only the fraction β of $d\rho_g$ newly generated glide dislocations is trapped in the material while the remainder is annihilated, then a newly generated dislocation density increment $d\rho_g$ causes a shear strain increment

$$d\gamma = glb\, d\rho_g = (gl\, b/\beta)\, d\rho \approx (gb/\beta)\, (d\rho/\sqrt{\rho}) \qquad [4]$$

However, according to Eq. [1],

$$d\rho/\sqrt{\rho} = (2/\alpha Gb)\, d\tau \qquad [5]$$

for

$$d\gamma \cong (2g/\alpha G\beta)\, d\tau \qquad [6]$$

Thus, the specific work-hardening coefficient on account of rising dislocation density becomes

$$\Theta/G = (d\tau/d\gamma)/G \cong \alpha\beta/2g \qquad [7]$$

The low work-hardening rate in stage I results from a large value of g, as mutual trapping only occurs among few and far between dislocations, while the "dislocation retention parameter," β, is unity and empirically α is always near one-third within a factor of 2 or so. Once the material is essentially filled with glide dislocation structures, as is the case in stages II and III, g is likely to be about 30, again within a factor of 2 or so,[28] so that one obtains, very roughly, $\Theta/G \approx \beta/200$. The empirical value of $\Theta/G \cong 1/200$ for stage II therefore suggests that $\beta \approx 1$, *i.e.*, that the large majority of glide dislocations is trapped in stage II. However, based on similitude in conjunction with Eq. [7], the rapidly decreasing work-hardening rate in stage III can hardly be due to a change in α and/or g but must be caused by an increasing rate of mutual dislocation annihilation, *i.e.*, diminishing β. And indeed it may be concluded that, except in stage I when g is large and somewhat variable, the dependence of Θ/G on flow stress mirrors the dependence of β on flow stress and thus dislocation density.[18,20,28]

D. *LEDSs and the LEDS Principle*

Structures of the universally expected kind, *i.e.*, in which near-neighbor dislocations mutually screen their shear stresses to the level of τ_0 so that Eq. [2] applies, have been named LEDS,[29] and the contention that in plastic deformation glide dislocations mutually trap into LEDSs (*i.e.*, not any of the obstructionist structures assumed to cause work-hardening in competing models) has been called the "LEDS principle." To the author's best knowledge and belief, in any type of deformation under any conditions short of explosive shock loading,[30] there are no exceptions to the LEDS principle.[31] In fact, as was already pointed out in Section I–D of Part I, the many studies on the development of dislocation cell microstructures of recent years, among them References 60 through 68 of Part I, essentially prove the LEDS principle.

E. *The LEDS Hypothesis*

Some years after Reference 20, the LEDS theory (then still under the name of "mesh-length theory") was expanded to include what is now known as the LEDS hypothesis,[32] namely, in its most recent formulation that "among all structures which in principle would be accessible to the system, always those microstructures are formed which are in equilibrium with the applied stresses *and* minimize the stored energy of the system." At the time,[32] it was phrased differently, namely, in conjunction with the principle of similitude as follows.[33] "The actual configuration must represent the configuration of minimum free energy among all configurations accessible to the system under the given conditions of straining such as speed of straining, temperature, stacking fault energy, *etc*. Therefore the same configuration but on a scale inversely proportional to the difference between the applied stress and τ_0, is also in mechanical equilibrium at any other stress (N.B. the stresses due to dislocations are inversely proportional to the distance from the dislocation axis) and thus is the configuration of minimum free energy among all accessible configurations. This is the principle of similitude. As indicated already, the actual configuration realized must be a function of the conditions of straining. However, even if these are kept constant, similitude must break down eventually. Either the mechanical stability of the configuration breaks down . . . Alternatively, additional configurations with smaller free energy may become accessible because, for example, it becomes possible to drag jogs effectively, or cross slip, conservative climb or climb becomes easy."

F. *Dislocation Cell Size Refinement in Stages III and IV*

The last consideration in the preceding quote, regarding the cessation of similitude with one form of LEDS and the consequent establishment of another LEDS which in turn is subject to similitude, explains the origin of work-hardening stages. Since these early insights,[20,28–32] the theory has been greatly expanded and refined. Herein the next great step forward was the solution to a previously perplexing question regarding the cell structures in stages III and IV: Why, with the specific dislocation line energy decreasing with angular misorientation of cell walls (Eq. [2]), do cells not grow indefinitely but continue to refine with flow stress in accordance with the widely observed empirical relationship for the average cell diameter

$$D = KGb/(\tau - \tau_0) \qquad [8]$$

where K is on the order of ten? The cause was found only comparatively recently. It is the presence of strain gradients, of average wavelength D, which are unavoidable in any mosaic block structure of mutually misoriented dislocation cells.[34] With the maximum shear stress due to those strain gradients limited by the flow stress, the associated elastic strain energy is

$$U_{el} \approx \tau^2/6G \quad [9]$$

independent of cell size. Following the LEDS hypothesis, those strain gradients will be replaced by corresponding dislocation walls; *i.e.*, the structure will be "polygonized," once their energy becomes smaller than U_{el}. That wall energy is, with Eq. [2],

$$U_P \cong 0.1\ (b/D)\tau \ln\ (G/\tau) \quad [10]$$

and for cubic cells the condition of $U_P \leq U_{el}$ leads to Eq. [8] with[34]

$$K \cong 0.9 \ln\ (G/\tau) \quad [11]$$

For typical values of $G/\tau \cong 700$, this yields $K = 6$, to be compared with Raj and Pharr's impressive analysis of available data[35] in support of Eq. [8], according to which $K \cong 8$ but with a tendency to decrease with rising strain.

G. *The Work-Hardening Stages: "Wavy" and "Planar" Glide—Cells vs Taylor Lattices*

Stage II begins at the end of stage I, if any, with a "LEDS transformation" that is rather well understood.[18,21] It leads to the planar dislocation structures of stage II, whereas stages III and IV are characterized by dislocation cells of manifold shapes. Basically, the stage II structures are tilt walls parallel to the primary glide plane with their tilt axis normal to the primary Burgers vector. In the so-called "wavy glide materials," including almost all pure cubic metals as well as many alloys, they assume the form of "carpets" of densely spaced dislocations that are separated by almost dislocation-free slabs of material. By contrast, in "planar-glide materials," the stage II tilt walls are composed of well-separated dislocations that are more or less evenly spaced so as to form (often rather complex) "Taylor lattices."

Both of these planar structures, aligned with the primary glide plane, bespeak low or no three-dimensional dislocation mobility. This explains why β remains near unity and the work-hardening coefficient remains high and nearly constant at about $G/200$ in stage II (Section C), since a modicum of three-dimensional mobility is required for significant mutual dislocation annihilation.

Planar-glide materials mostly comprise fcc alloys with a low stacking fault energy for which α-brass is prototypical.[36] In these, the flow stress is controlled by dislocation bypassing[18,21,28,36] instead of supercritical dislocation bowing as in dislocation cell structures Equations [1] and [4] through [7] are readily adapted to this case; too, and Eq. [7] continues to apply also in planar-glide but with a modified, numerically rather similar value of α.[18,21]

Stage III begins when those planar-glide structures of tilt walls more or less parallel to the primary glide plane break down and give place to a cell structure. This stage II/stage III LEDS transformation occurs at the so-called τ_{III} stress that decreases with straining temperature. In wavy-glide materials the stage II carpets readily transform into the more or less equiaxed cell structure of stage III, and its beginnings are occasionally seen in micrographs, *e.g.*, in Figure 3(a) of Reference 18. According to a great deal of research by Seeger and Haasen and co-workers, the temperature and speed dependence of τ_{III} proves the involvement of cross-slip herein, and this is also in agreement with the LEDS theory.[18,21] However, in planar-glide materials the corresponding LEDS transformation, namely, from Taylor lattices to cell structures, is not easily accomplished. It is therefore delayed to higher stresses or may not take place at all, especially below $T_M/2$. However, with that LEDS transformation, "planar-glide" materials at the same time assume the wavy-glide mode.

In the stage III to stage IV LEDS transformation, the equiaxed stage III cells give place to cell shapes that tend to be homologous with the imposed shape changes. The associated free energy reduction arises through "relaxed constraints," somewhat analogous to the formation of DBs, *i.e.*, buying reduced mismatch stresses with a moderately increased dislocation cell wall area at a total energy reduction. The microscopical details are very complex, and the reader is referred to References 18 and 21 for further information and references to the relevant literature.

H. *The Shape and Temperature Dependence of the Work-Hardening Curve*

In determining the shape of the stress-strain curve and its temperature dependence, we make use of the following facts: (1) The stress-strain curve mirrors the value of β as explained (Section C); (2) significant deviations of β from unity are overwhelmingly due to mutual dislocation annihilations; (3) mutual dislocation annihilations are possible only among locally parallel dislocation segments; (4) they occur on account of mutual attractions among closely neighboring dislocations, driven by the dislocations' self-stresses that fall off as $1/r$ from their axes; and (5) the average approach among dislocations is proportional to $l \cong 1/\sqrt{\rho}$, which in turn is inversely proportional to the flow stress *via* Eq. [1]. Having thus established a connection between β and the flow stress and thereby the stress dependence of the work-hardening coefficient, Θ, the work-hardening curve, $\tau(\gamma)$, is found by integration. The result is a linear decrease of β with the flow stress. The thereby implied linear dependence of the work-hardening coefficient with flow stress in turn yields the Voce curve for the work-hardening curve, in excellent agreement with observation for stage III.

Considering further the effect of thermal activation on mutual annihilation, one finds for the curves of Θ/G as a function of stress a slow increase of the slope with increasing deformation temperature and a regular displacement of the intercepts of the Θ/G curves with the stress axis, *i.e.*, of the stress of projected zero work hardening. All these results are in fine agreement with experimental data in the literature.[18,21] In a nutshell, this is the LEDS theory. A full exposition of it, including also a theoretical interpretation of the mechanism of recovery, Hall-Petch and alloy hardening, slip line structures, strain rate dependence of the flow stress, and other topics, may be found in the recent survey articles.[18,21] Thus, the LEDS theory gives an account for all of the major phenomena of dislocation-based crystal plasticity. Even so, many details still await the input of computer modeling.

III. THE ROLE OF DEFORMATION BANDS IN THE LEDS THEORY

A. *Incorporating DBs into the LEDS Theory*

Quite deliberately, in order not to contaminate the LEDS theory with dubious assumptions and render it unreliable, it

has throughout been developed from basic principles with a minimum of model making. This may already have become apparent in the preceding explanations and is based on the "theory of theories." Namely, the probability that any theory is correct, P_c, is at best the product of the corresponding probabilities of all of the necessary underlying assumptions. The very detrimental effect of using any but the most secure assumptions becomes apparent when considering that already three plausible assumptions, each with an 80 pct probability of being correct, yields $P_c = 0.8^3 = 0.52$; *i.e.*, a fifty-fifty chance of the theory being incorrect and presumably not worth publishing.

In the LEDS theory, therefore, all but the most secure assumptions have been strenuously avoided. As a result, it is sufficiently nonspecific to be broadly applicable and permits the introduction of deformation banding by simple expansion without change in the preceding status, as follows.

Restrictively, the only two requirements for DBs to fit harmoniously into the theory are (1) that DB boundaries are LEDSs as defined previously *i.e.*, are dislocation rotation boundaries, (nearly) obeying Frank's formula,[37] and (2) that they are at least predominantly formed of mutually trapped glide dislocations. Both of these features are already established for all previously considered kinds of dislocation walls,[38] and they have been additionally verified in the already mentioned relevant articles cited in Part I.

The required expansion of the LEDS theory consists of adding the mismatch stresses due to deviations from homologous deformation to the previously recognized driving forces acting on dislocations. In the past, the decisive driving forces were recognized as the applied stresses and the dislocation-dislocation interactive stresses in accordance with Eq. [2]. The discussed mismatch stresses are much smaller than these and therefore were overlooked. Even so, this expansion of the theory is very valuable, as further discussed in Section B.

B. *"Geometrically Necessary" and "Redundant" Dislocations*

In the past, the role of strain gradient energies associated with random dislocation wall stresses has been recognized in connection with cell size refinement,[34] as already discussed (Eqs. [8] through [11]), as well as, somewhat rudimentarily, also in texture formation.[39] These considerations must now be expanded and refined. Namely, as a consequence of the LEDS hypothesis, a peculiar synergy exists in regard to the roles of "redundant dislocations" *vs* "geometrically necessary" dislocations, or "nonredundant" dislocations in Weertman's terminology,[40,41,42] *i.e.*, those that are associated with strain gradients. The critical consideration herein is the dual role of glide dislocations trapped into cell walls and/or DB boundaries (Section 18 of Reference 18). On the one hand, they are "incidental,"[43] since they were simply trapped into LEDSs out of which the applied stresses could not free them. On the other hand, they are "geometrically necessary,"[43] since geometrically, all glide dislocations are the boundaries across which the shear displacement changes by their specific Burgers vector, **b**, so that a wall of trapped glide dislocations necessarily defines the mutual lattice rotation specified by Frank's formula.[37]

In a somewhat unexpected manner, this dual role of trapped glide dislocations is intertwined with the LEDS hypothesis. This is due to the connection between the dislocation spacing in cell walls and the associated lattice rotation angle

$$\phi \cong b/l \qquad [12]$$

that was already used in deriving Eq. [10]. Namely, since all glide dislocations contribute to the strain in accordance with Eq. [4], there is no limit to the ϕ values. Rather, the larger the misorientation angles, the more favorable in terms of Eq. [2] are the energetics, subject only to dislocation cell refinement, which during straining continuously generates new cell walls with low ϕ values (Section II–F).

It follows, then, that a widespread of cell wall misorientations is to be expected based on the LEDS hypothesis. This accounts for the fact that "cell blocks" form with significant angular misorientations among each other, themselves being subdivided into "ordinary" cells, *e.g.*, as observed first by Bay *et al.*,[44,45,46] It follows also that the primary relationship is between stress and dislocation cell size, (Eq. [8]), whereas Eq. [1] is only approximate. This causes α to decrease with stress in addition to and beyond that decrease of α which results from the log term in Eq. [2] [28,32]. The aforementioned complex synergy results because the evolving misorientations due to trapped glide dislocations cause changes in Taylor factors and hence slip system selections; these in turn cause additional strain gradients of the kind also associated with DBs, and these again give rise to modifications of the pattern of new cell wall generation and their maturing. Certainly, on a microscopic scale, the discussed synergy, in conjunction with the rise of flow stress with increasing number of locally simultaneously acting slip systems, is the root cause of the evolution of texture components,[18,21,39] in agreement also with the various pertinent articles cited in Part I. Presumably it will be quite difficult to obtain predictive relationships in this process. Yet such relationships will constitute a decisive advance in the evolution of improved constitutive equations.

IV. NEAR PROOF OF THE LEDS HYPOTHESIS BASED ON DBs

While none of the considerations in Sections II and III pose any problem in the LEDS theory but rather are in excellent accord with it, they reveal the previously unrecognized role played by the longer-range stresses about volumina operating with different slip system selections. That role had escaped attention until now because in wavy-glide, the impact of those stresses on the stress-strain curve and work hardening is minor. For most strength properties in wavy-glide, it does not matter much whether the segregation of one to three slip systems in lieu of five takes the form of evident DBs or of smaller, less regular volume elements. As already explained, this is for the reason that those longer-range stresses that are minimized through evident deformation banding are rather small. However, the very fact that they nonetheless have the potential of greatly affecting the slip morphology, namely in the form of DBs, can be used for a near proof of the LEDS hypothesis. To this end, we shall consider how far from or near to thermodynamic equilibrium DBs are formed, recognizing that DBs represent but one part of the total LEDSs.

To begin with, W_{stored}, the free energy density stored in LEDSs at flow stress τ, is largely independent of detailed

morphology. Specifically, Eqs. [1] and [2], with $\alpha \cong 1/3$, ln, $(1/b\sqrt{\rho}) \cong 6$. Poisson's ratio $\nu = 0.3$, and a correction factor of $M_r = 2$ to take account of imperfect equilibration and redundant dislocation within cell walls, yield

$$W_{\text{stored}} \cong M_r \rho G b^2 \{(1 - \nu/2)/[4\pi(1 - \nu)]\} \ln (1/b\sqrt{\rho}) \cong 6\tau^2/G = 6G\ \gamma_{el}^2 \quad [13]$$

where γ_{el} is the elastic shear strain. This compares with W_{el}, the elastic strain energy density because of the applied flow stress, *e.g.*, in tension or compression, with M the Taylor factor,

$$W_{el} = (1/2)\ \varepsilon_{el}^2 E = (1 + \nu)\ G\gamma_{el}^2/M^2 \cong 0.2\ G\gamma_{el}^2 = W_{\text{stored}}/30 \quad [14]$$

Meanwhile, for all but small plastic strains, γ_{pl}, the work-input density is

$$W_{in} \cong (2/3)\ \tau\gamma_{pl} = (2/3)\, G\gamma_{el}\gamma_{pl} = (1/9)\, W_{\text{stored}}(\gamma_{pl}/\gamma_{el}) \quad [15]$$

Hence, since typically γ_{pl}/γ_{el} is a large number, plastic deformation is one of nature's most effective ways of converting mechanical energy into heat.

In order to assess how far from equilibrium LEDSs might be, it is useful to compute the temperature rise if the stored energy were converted into heat, *i.e.*,

$$\Delta T_{\text{stored}} = W_{\text{stored}}/cd_m \cong 6G\gamma_{el}^2/(cd_m) \quad [16]$$

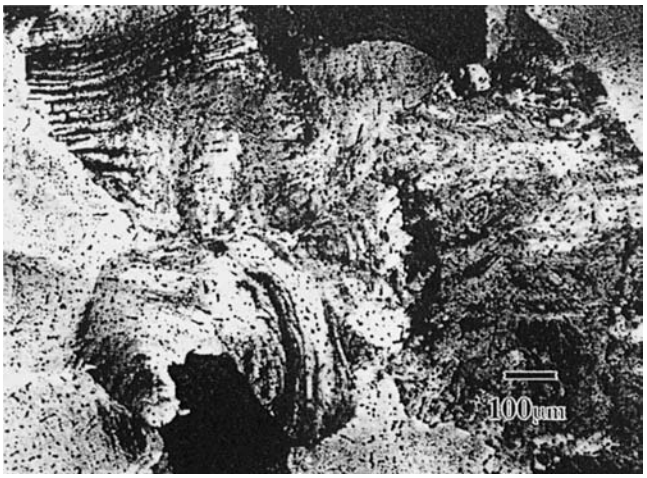
where c is the specific heat and d_m the mechanical density. For most metals of technological interest, the numerical value of G/cd_m at room temperature ranges about 15,000 °C, and $\gamma_{el} = 1$ pct is near the upper limit in the deformation of technological materials. For this, Eq. [16] yields

$$\Delta T_{\text{stored, }\gamma el} \cong 6 \times 10^{-4}\ G/cd_m \cong 9\ ^\circ\text{C} \quad [17]$$

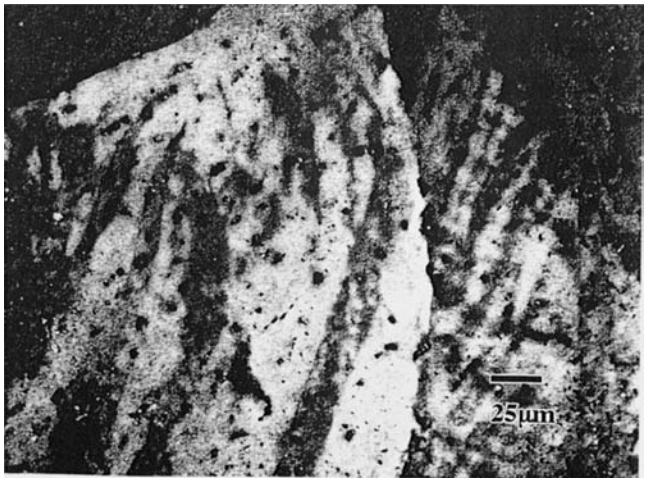
within a factor of 2 or so. Meanwhile γ_{pl} is liable to exceed unity, and the work-input density would heat the material through a few hundred degree Centigrade.

These estimates do not support the assumption of SODS modeling that plastic deformation occurs *far* from equilibrium. Yet, 9 °C heating amounts to roughly $T_M/100$ and could still leave room for argument as to what "far" means in this context. In fact, until recently, the last remaining obstacle that was believed to stand in the way of the universal acceptance of the LEDS theory and corresponding abandoning of the SODS approach was the perceived impossibility to prove the LEDS hypothesis. It was considered that it is the nature of fundamental hypotheses that they can never be proven absolutely. And in the case of the LEDS hypothesis, the problem was compounded through the difficulty of accurately determining the free energies of the vast numbers of intrinsically possible LEDSs.

Fortunately, this position is fundamentally changed when considering the energy density differences that cause deformation banding, *e.g.*, as in Figures 8 through 12 of Part I and Figure 1 herein. Since the energy due to the band boundaries decreases inversely with the average bandwidth, w, and the energy due to the misfit stresses, being principally concentrated at their ends, rises approximately linearly with w, the



(*a*)



(*b*)

Fig. 1—(*a*) and (*b*) Optical micrographs of the etched midplane of Al-Cu-Si after uniaxial compression at ambient temperature to −69 pct true strain at two different magnifications, as indicated (Fig. 10 of Kulkarni *et al.*[50]).

energy of the banding is due in nearly equal parts to those two contributions.[23,24,47,48] Seeing that the specific band boundary energy, Γ, is much more readily estimated than the energy of the end-stresses, the energy density due to deformation banding is therefore most conveniently written as

$$W_{DB} \cong 2\Gamma/w \quad [18]$$

The ideal boundary energy may be found from Eqs. [1] and [2] with Eq. [12], but actually (following the early and since then well-confirmed[39] observations by Bailey and Hirsch[49]), Γ is higher by the correction factor of $M_r \cong 2$ because of point defects and redundant dislocations, which was already introduced in Eq. [13]. Thus, for $\nu = 0.3$, as before,

$$\Gamma \cong M_r\ \phi\ Gb\ \{(1 - \nu/2)/[4\pi(1 - \nu)]\} \ln (1/\phi) \cong (0.1\ M_r\ \phi\ Gb/w) \ln (1/\phi) \quad [19]$$

The maximum value of Γ occurs at $\phi = 1/e = 21.08$ deg, at which point with $M_r = 2$, it is

$$\Gamma_{\max} \cong 0.074Gb \cong 0.028Eb \quad [20]$$

In general, $\Gamma = f\Gamma_{\max}$ with $f < 1$, not only because angular

misorientations fall short of $\phi = 21.08$ deg but also because of the cell walls of the underlying mosaic block structure.[23] Hence, the equivalent temperature rise of deformation band formation is

$$\Delta T_{DB} \leq 2\,\Gamma_{\max}/wcd_m \cong 0.148Gb/wcd_m \cong 0.057Eb/w \qquad [21]$$

Table I presents some numerical values obtained for DBs in Figures 8 through 11 of Part I and Figure 1 of the present article. From these, it will be seen that ΔT_{DB} in the selected cases falls mostly below 1/100 of a degree Centigrade and typically amounts to only one-tenth of ΔT_{el} on account of the elastic energy due to the applied flow stress. In line with the exposition in Section I, this proves for the particular DBs considered that they are very close to equilibrium indeed. Seeing that the DBs are but part of the whole complex LED structure in the samples, this rules out the SODS assumption that dislocation structures form far from equilibrium. This conclusion is further reinforced by the occasional symmetry and exquisite regularity of deformation banding, which cannot be explained by "self-organization" caused by dynamic processes. An impressive example from our own collection of micrographs has been presented in earlier publications.[23,50] based on SS Kulkarni's Masters Thesis.[51] Perhaps even more impressive is Figure 2 extracted from Reed and McHargue's research on DBs in extruded aluminum single crystals.[52] The remarkable symmetry and refinement of the DB structures in this case bespeak the approach to equilibrium configurations.

V. DEFORMATION BAND MORPHOLOGY

As already discussed, the morphology of DBs will very nearly minimize the sum of the extra elastic mismatch strain energy (W_a) plus the energy of the boundaries between the bands (W_B). If, then, the flat parallel sides of an ideal family of bands of length L fit together with negligible mismatch, the elastic mismatch strain energy will be concentrated within a zone of length $\lambda_a w$ at the ends of the bands. Here,

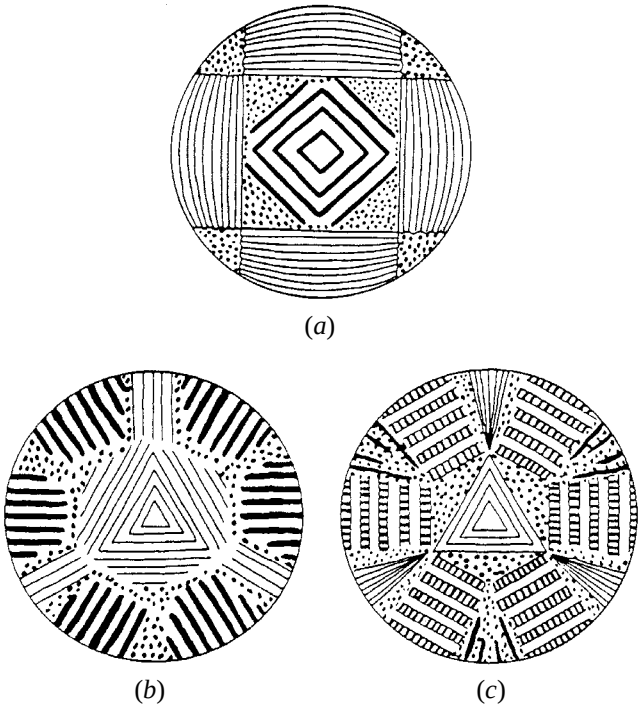


Fig. 2—Schematic of the deformation band structure in the cross sections of initially symmetrically oriented, extruded aluminum crystals. The crystals exhibited the well-known double ⟨111⟩ (majority) plus ⟨100⟩ (minority) fiber texture, but both texture elements are of rather imperfect definition because of the dislocation cell structure in them. The minority ⟨100⟩ oriented volume elements are indicated in heavy black as embedded in the white-background ⟨111⟩ matrix, while DB boundaries free of ⟨100⟩ texture component are given as fine dark lines. The ⟨100⟩ texture elements predominantly form thin seams along the boundaries of ⟨111⟩ oriented deformation bands, as indicated by the heavy dark lines, and also as small colonies within ⟨111⟩ matrix, indicated by stippling. Initial orientations are (*a*) [100] as extruded at room temperature, and (*b*) [111] as extruded at room temperature, and (*c*) at liquid nitrogen temperature. Note the remarkable symmetry and regularity of the structures, with the finest ⟨100⟩ bands only one single dislocation cell wide (Fig. 11 of Reed and McHargue[52]).

Table I.

L (mm)	w (mm)	ε	γ_{el} ($M = 2.5$)	σ (MPa)	$\Gamma = f\Gamma_{\max}$ (N/m)	W_{in} (MJ/m^3)	W_{el} (MJ/m^3)	W_{stored} (MJ/m^3)	W_{DB} (MJ/m^3)	ΔT_{in} (°C)	ΔT_{el} (°C)	ΔT_{stored} (°C)	ΔT_{DB} (°C)
Al-0.5 pct Cu, Fig. 8 of Part I, top middle left, deformed at −196 °C, assumed $f = 3/4$													
2.8	0.09	−0.47	0.39 pct	110	0.41	37	0.082	2.5	0.0091	15.5	0.034	1.04	0.0038
Al-0.5 pct Cu, Fig. 9 of Part I, center middle right, deformed at −196 °C, assumed $f = 1$													
4.6	0.12	−0.68	0.43 pct	120	0.56	61	0.10	3.0	0.0094	25.6	0.042	1.25	0.0039
Al-0.5 pct Cu, Fig. 10 of Part I, center lower right, deformed at room temperature, assumed $f = 1$													
0.88	0.10	−0.72	0.38 pct	108	0.56	58	0.083	2.3	0.011	24.4	0.035	0.97	0.0046
Al-0.5 pct Cu-1 pct Si, Fig. 11 of Part I, center, deformed at room temperature, assumed $f = 3/4$													
0.41	0.035	−0.69	0.61 pct	173	0.41	89	0.20	5.3	0.023	37.4	0.087	2.2	0.0097
Al-0.5 pct Cu-1 pct Si, Fig. 1 of this article, average, deformed at room temperature, assumed $f = 3/4$													
0.18	0.02	−0.69	0.61 pct	173	0.41	89	0.20	5.3	0.040	37.4	0.087	2.2	0.017

Stored energy densities (W_{xx}) and temperature rises ($\Delta T_{xx} = W_{xx}/cd_m$, with $c = 880$ J/kg °C the specific heat and $d_m = 2700$ kg/m^3 the mechanical density) that would result if the stored energy were converted into heat, for five different compressed aluminum alloy samples as listed. The symbols designate the following, respectively: L = average DB length, w = average DB width, ε = true compression strain, γ_{el} = elastic shear strain inferred from σ = measured compression stress, assuming a Taylor factor of $M = 2.5$ (compare Figs. 1 through 3 of Part I), Γ = specific DB boundary energy (Eqs. [19] and [20]), W_{in} = plastic work input per unit volume (graphically derived from Figs. 13 and 14 of Part I), $W_{el} = 0.2G\gamma_{el}^2$ = energy density due to elastic strain while under load (Eq. 14), W_{stored} = inferred energy stored in the dislocation cell structure (see eq. 13), and $W_{\text{DB}} = 2\Gamma/w$ the energy density comprised in the DB structure.

at the moment of band formation at flow stress τ_{DB}, the shear stress will rise, in general, from τ_{DB} to $\tau_{DB}\,(1 + \delta_a)$, at an associated mismatch energy density of

$$W_a = (2\lambda_a w/L)\ \tau_{DB}^2\,[(1 + \delta_a)^2 - 1]/2G \cong (2\lambda_a \delta_a w/L)\ \tau_{DB}^2/G \quad [22]$$

whereas the energy density due to the band boundaries is

$$W_B \approx \Gamma/w \quad [23]$$

The energy minimum is found from $d\,(W_a + W_B)/dw = 0$ at

$$w = \{(\Gamma LG)/[\lambda_a\,(2\delta_a + \delta_a^2)\ \tau_{DB}^2]\}^{1/2} \cong \{\Gamma LG/[2\lambda_a \delta_a\ \tau_{DB}^2]\}^{1/2} \quad [24]$$

Since we are not now concerned with finding conservative estimates as in the previous section, and in fact the deformation band boundary energy is reduced through the presence of the ordinary cell walls,[23] the best value for Γ here is

$$\Gamma \cong \Gamma_{max}/2 \cong 0.035Gb \quad [25]$$

One will therefore expect from Eqs. [24] and [25] that the parameter

$$\xi = [\lambda_a(2\delta_a + \delta_a^2)] \approx 0.035\ b(L/w^2)(G/\tau_{DB})^2 \quad [26]$$

is nearly constant. This relationship was tested on the only too restricted number of suitable micrographs from the literature and our own collection.[23] An average of $\xi \cong 0.8$ was found, which is compatible with $\xi = 1.2$ derived from the best theoretically expected values of $\delta_a \cong 1$ and $\lambda_a \cong 0.4$.

VI. DEFORMATION BANDS AND TEXTURE MODELING: REPLACING THE TAYLOR MINIMUM WORK CRITERION WITH THE MINIMUM FREE ENERGY CRITERION

The derivation of the equilibrium value of w^2/L in Eqs. [22] through [24] follows in outline the thoughts of Chin and Wonsiewicz[47] and the more recent and much more detailed theoretical considerations of Lee *et al.*,[48] albeit with one important modification: In the framework of the LEDS hypothesis, it is the energy density in the loaded state, or in other words *the free energy* at the moment of DB formation, that is minimized. By contrast, Chin and Wonsiewicz and Lee *et al.*, in the continuing tradition of texture modeling, minimized *the plastic work input, i.e.*, W_{pl}.

Numerically, the difference in these two approaches is as follows: For the data of w^2/L given by Lee *et al.* for their DBs in rolled copper of two widely different grain sizes,[48] Eq. [26] renders $\xi = 1.9$ and 0.63, respectively,[23,50] in satisfactory agreement with the (admittedly rather uncertain) best theoretical value of $\xi = 1.2$ already given at the end of Section III. By contrast, the equation derived by Lee *et al.* from Taylor's minimum plastic work principle agrees with those same data only if $\Gamma = 0.14Gb$ is assumed, an obviously much too high value (compare Eq. [25]). The numerical test is thus decidedly in favor of the LEDS hypothesis. More importantly, the plastic work minimization theorem makes no physical sense: How could the structure of a material at constant temperature possibly depend on how much work was done on it when 90 pct and more thereof was converted into heat? Moreover, the physical reason for the Taylor minimum work criterion is unaffected by considering free energy instead of plastic work input, since its function is simply to always let the most highly stressed slip systems operate. In conclusion, then, in future texture modeling, the Taylor minimum plastic work criterion must be replaced by the minimum stored energy density criterion.

VII. SUMMARY AND CONCLUSIONS

1. For historical reasons that pertain more to sociology than science, protracted controversies in the area of work-hardening theory have delayed the development of technologically useful constitutive equations.
2. They may now be based on the LEDS theory as it is fully consistent with the basic laws of physics and comprehensively explains all major facets of metal plasticity with a minimum of assumptions or model making.
3. In fact, the LEDS theory is but a special case of the more general LES hypothesis according to which plastic work done on materials generates states of order that are near thermodynamical equilibrium and that this constitutes one of nature's means to generate order.
4. The observations on DBs discussed in Part I are readily incorporated into the LEDS theory.
5. These observations further permit a test of the LEDS hypothesis as to the degree to which actual dislocation structures approach thermodynamical equilibrium.
6. Whereas the total work input through the deforming tractions is commonly equivalent to tens of degrees Centigrade heating of the material and more, the energies stored in the dislocation structures and DBs would heat the material through only about 1 °C and 0.01 °C, respectively.
7. That means that the dislocation structure is finely tuned so as to approach free energy minimization, *i.e.*, thermodynamical equilibrium, to a remarkably close degree.
8. This result virtually proves the LEDS theory but is in stark conflict with the assumption of SODS modeling that plastic deformation occurs far from equilibrium.
9. With the demonstrated success of the LEDS theory, the way is now open to begin the development of constitutive equations based thereon.
10. Accordingly, in future texture modeling, the Taylor criterion of minimum work input should be replaced by the criterion of minimum free energy of the deformed material while under stress.

ACKNOWLEDGMENTS

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TABLE OF SYMBOLS

b	magnitude of the Burgers vector
$\mathbf{b}$	Burgers vector
c	specific heat
d_m	mechanical density
$d\gamma$	shear strain increment caused by $d\rho_g$ newly generated glide dislocations

$d\rho_g$ increment of newly generated glide dislocations
D average dislocation cell size in stages III and IV
DB "regular" deformation band, *i.e.*, a slab-like volume element with a selection of active slip systems differing from that in neighbor bands
$E = \sigma/\varepsilon$ Young's modulus, the ratio of tensile stress to tensile strain
$f = \Gamma/\Gamma_{max}$ correction factor taking into account the presence of cell walls with ϕ values of $<1/e$
g the glide path parameter, being the mean free dislocation path length in units of the mean dislocation link length or spacing, l
G shear modulus
K numerical parameter in the order of ten, which links the cell size to the flow stress (Eq. [8])
$l \cong 1/\sqrt{\rho}$ average dislocation link length or distance between dislocations in LEDSs
L average deformation band length
LEDS dislocation structure within which near-neighbor dislocations mutually screen their shear stresses to the level of τ_0
$M_r \cong 2$ factor by which the dislocation density in cell walls exceeds the geometrically necessary
P_c probability that a theory is correct according to the theory of theories
SODS dislocation structure expected from dislocation self-organization far from equilibrium in accordance with Prigogine's thermodynamics of energy-flow-through systems
T_M absolute melting point temperature
U_D average dislocation line energy in a LEDS (Eq. [2])
U_{el} elastic strain energy associated with strain energy gradients in a mosaic block structure
U_P energy density associated with new cell walls being generated on account of U_{el}
w average deformation bandwidth
W_a energy density on account of strain mismatch at DB ends
W_B energy density on account of DB boundaries
W_{DB} energy density on account of the presence of regular deformation bands $= W_a + W_B$
$W_{el} = \tau^2/G$ elastic strain energy density on account of applied flow stress, τ
W_{in} work input density on account of plastic strain application of γ_{pl}
W_{stored} energy density stored in the from of LEDSs
$\alpha \cong 1/3$ the flow stress parameter, linking the flow stress to the dislocation density (Eq. [1])
β the dislocation retention parameter, being the fraction of newly generated glide dislocations that is trapped in the LEDS structure
$\gamma_{el} = \tau/G$ elastic shear stain at flow stress τ
γ_{pl} plastic strain between the undeformed state and application of flow stress τ
Γ specific deformation band boundary energy
Γ_{max} maximum value of Γ, attained for $\phi \geq 1/e = 21.08$ deg (Eq. [20])
δ_a fractional increase of the local stress at deformation band ends
ΔT_{DB} temperature rise if the energy stored in deformation bands, W_{DB}, were converted into heat
ΔT_{stored} temperature rise if the stored energy, W_{stored}, were converted into heat
λ_a width of zone of enhanced stress, in units of w, on account of strain mismatch at DB ends
Θ work-hardening coefficient
ν Poisson's ratio.
ϕ lattice rotation angle across a dislocation rotation boundary, *e.g.*, a cell wall or DB boundary
ρ dislocation density
τ flow stress, critical resolved shear stress
τ_{DB} flow stress at the point of deformation band formation
τ_0 friction stress, resolved shear stress required to move a straight, isolated dislocation
ξ measurable parameter linking various values relevant to DBs, theoretically expected to be not far from unity (Eq. [26]).

REFERENCES

1. G.I. Taylor: *Proc. R. Soc. (London)*, 1934, vol. 145, pp. 362-87, 388-404, and 405-15.
2. R. Becker: *Phys. Z*, 1925, vol. 26, pp. 919-25.
3. R. Becker: *Z Phys.* 1925, vol. 3, p. 185.
4. R. Becker: *Z. Technol. Phys.*, 1926, vol. 7 p 547.
5. R. Becker and W. Boas: *Metallwirtschaft*, 1929, vol. 8, pp. 317-21.
6. R. Becker and E. Orowan: *Z. Phys.*, 1932, vol. 79, pp. 566-72.
7. D.L. Holt: *J. Appl. Phys.*, 1970, vol. 41, pp. 3197-3201.
8. M.R. Staker and D.L. Holt: *Acta Mater.*, 1972, vol. 20, pp. 569-74.
9. E.C. Aifantis: *J. Eng. Mater. Technol.*, 1984, vol. 106, pp. 326-30.
10. E.C. Aifantis: *Mater. Sci. Eng.*, 1986. vol. 81, pp. 563-74.
11. E.C. Aifantis: *Patterns, Defects and Microstructures in Non-Equilibrium Systems*, NATO ASI Series, Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1987, pp. 221-39.
12. D. Walgraef and E.C. Aifantis: *Int. J. Eng. Sci.*, 1985, vol. 23, pp. 1351-58, 1359-64, and 1365-72.
13. *Patterns, Defects and Microstructures in Non-Equilibrium Systems*, D. Walgraef, ed., NATO ASI Series, Martinus Nijhoff Publishers, Dordrecht, The Netherlands, 1987.
14. L.P. Kubin: in *Materials Science and Technology: A Comprehensive Treatment*, R.W. Cahn, P. Haasen, and F.L. Kramer, eds., VCH, Weinheim, 1993, vol. 6 pp. 138-90.
15. L.P. Kubin: in *Stability of Materials*, A. Gonis, P.E.A. Turchi, and J. Kudrnovsky, eds., Plenum Press, New York, NY, 1995, pp. 99-135.
16. B. Devincre and L.P. Kubin: *Mater. Sci. Eng.*, Special Issue of ICSMA Conf., Prague, 1997, vol. A234, pp. 8-14.
17. D. Kuhlmann-Wilsdorf: *Metall. Trans. A*, 1985, vol. 16A, pp. 2091-2108.
18. D. Kuhlmann-Wilsdorf: *Mater. Res. Innov.*, 1998, vol. 1, pp. 265-97.
19. G. Batchelor: *The Life and Legacy of G.I. Taylor*, Cambridge University Press, Cambridge, United Kingdom, 1996.
20. D. Kuhlmann-Wilsdorf: *Trans. TMS-AIME*, 1962, vol. 224, pp. 1047-61.
21. D. Kuhlmann-Wilsdorf: *Phil Mag.*, 1999, vol. A79, pp. 955-1008.
22. D. Kuhlmann-Wilsdorf: *Scripta Metall. Mater.*, 1997, vol. 36, pp. 173-81.
23. D. Kuhlmann-Wilsdorf: *Acta Metall. Mater.*, 1999, vol. 44, pp. 1699-712.
24. D. Kuhlmann-Wilsdorf and K.I. Winey: *J. Appl. Phys.*, 1999 vol. 85, pp. 6392-99.
25. P.S. Pinheiro and K.I. Winey: *Macromolecules*, 1998, vol. 31, pp. 4447-53.
26. C. Laird, Z. Wang, B.-T Ma, and H.-F. Chai: *Mater. Sci. Eng.*, 1989, vol. A113, pp. 245-65.
27. L. Llanes and C. Laird: *Mater. Sci. Eng.*, 1993, vol. A161, pp. 1-12.
28. D. Kuhlmann-Wilsdorf: *Mater. Sci. Eng.*, 1989, vol. A113, pp 1-41.
29. See foreword to *Low-Energy Dislocation Structures*, Proc. Int. Conf. on Low-Energy Dislocation Structures, Charlottesville, VA, M.N. Bassim,

W. A. Jesser, D. Kuhlmann-Wilsdorf, and H.G.F. Wilsdorf, Elsevier Sequoia, Lausanne, 1986; see also *Mater. Sci. Eng.*, 1986, vol. 81, pp. 1-574; 1987, vol. 86, pp. 19-92.
30. L.E. Murr and D. Kuhlmann-Wilsdorf: *Acta. Metall.*, 1978, vol. 26, pp. 847-57.
31. D. Kuhlmann-Wilsdorf: *Phys. Status Solidi*, 1987, (a)104, pp. 121-44.
32. D. Kuhlmann-Wilsdorf: in *Workhardening*, J.P. Hirth and J. Weertman, eds., Gordon and Breach, New York, NY, 1968, pp. 97-132.
33. D. Kuhlmann-Wilsdorf: in *Workhardening*, J.P. Hirth and J. Weertman, eds., Gordon and Breach, New York, NY, 1968, pp. 99-100.
34. D. Kuhlmann-Wilsdorf: *Scripta Metall. Mater.*, 1996, vol. 34, pp. 641-50.
35. S.V. Raj and G.M. Pharr: *Mater. Sci. Eng.*, 1986, vol. 81, pp. 217-37.
36. D. Kuhlmann-Wilsdorf, H.G.F. Wilsdorf, and J.A. Wert: *Scripta Metall. Mater.*, 1994, vol. 31, pp. 729-34.
37. F.C. Frank: *Pittsburgh Symp. Plastic Deformation of Crystalline Solids*, Report No. NAVEXOS-P-834, Office of Naval Research, Washington DC, 1950, p. 150; see also W.T. Read: *Dislocations in Crystals*, McGraw-Hill, Inc., New York, NY, 1953, pp. 181-86.
38. D. Kuhlmann-Wilsdorf: *Phys. Status Solidi*, 1995, vol. (a)149, pp. 225-41.
39. D. Kuhlmann-Wilsdorf: *Scripta Metall. Mater.*, 1997, vol. 36, pp. 173-81.
40. J. Weertman: *Proc. R. Soc. (London)*, 1989, vol. A425, p. 291; 1991, vol. A435, pp. 43 and 69.
41. J. A. Hurtado and J. Weertman: *Phil. Mag.*, 1993, vol. A68, p. 599.
42. J. A. Hurtado and J. Weertman: *Phys. status solidi*, 1995, vol. (a)149, pp. 173-86.
43. D. Kuhlmann-Wilsdorf and N. Hansen: *Scripta Metall. Mater.*, 1991, vol. 25, pp. 1557-62.
44. B. Bay, N. Hansen, and D. Kuhlmann-Wilsdorf: *Mater. Sci. Eng.*, 1989, vol. A113, pp. 385-97.
45. B. Bay, N. Hansen, and D. Kuhlmann-Wilsdorf: *Mater. Sci. Eng.*, 1992, vol. A158, pp. 139-46.
46. B. Bay, N. Hansen, D.A. Hughes, and D. Kuhlmann-Wilsdorf: *Acta Metall. Mater.*, 1992, vol. 40, pp. 205-19.
47. G.Y. Chin and B.C. Wonsiewicz: *Trans. AIME*, 1969, vol. 245, pp. 871-72.
48. C.S. Lee, B.J. Duggan, and R.E. Smallman: *Acta Metall. Mater.*, 1993, vol. 41, pp. 2265-70.
49. J.E. Bailey and P.B. Hirsch: *Phil. Mag.*, 1960, vol. 5, pp. 485-97.
50. S.S. Kulkarni, E.A. Starke, Jr., and D. Kuhlmann-Wilsdorf: *Acta Metall. Mater.*, 1998, vol. 46, pp. 5283-5301.
51. S.S. Kulkarni: Master's Thesis, University of Virginia, Charlottesville, VA, 1997.
52. R.E. Reed and C.J. McHargue: *Trans. AIME*, 1967, vol. 239, pp. 1604-12.